M. REDHEAD

MODELS IN PHYSICS

The paper is concerned with the role of theoretical models in physics. These are to be distinguished from analogue and representational models, from approximations and also from the technical use of the term in mathematical logic. Theoretical models arise in two situations: (1) When a theory is too complicated to apply exactly a simplified model may be employed. In this case models play the role of impoverished theories, an important ingredient being that we believe the description afforded by the model to be in point of fact false. (2) A theory may be incompletely specified by some general constraints or principles. A model may be proposed which fills in the details of the theory. In this case models serve to enrich theories. They remain models, as distinct from theories, insofar as they are not proposed, even tentatively, as true descriptions of reality. Models and theories can change their roles at different historical epochs. If a theory proposed as a true description is later shown to be false, it may still serve as a model for the new theory. Similarly if the model mechanism used to enrich an incompletely specified theory turns out to give successful predictions the model may come to achieve the honorific status of theory.

Three uses of models are distinguished,

1. The articulation of theories:

Models can be used to see what kinds of qualitative behaviour a complicated theory might have. This may lead to successful investigation as to whether the complete theory also exhibits this behaviour - without the use of models we might not know what to look for in the complicated theory.

2. The heuristic role of models:

If a theory is too complicated to be applied exactly to some group of phenomena, a model may be proposed which is more tractable. This model may have some property which serves to explain some aspect of the phenomena. We do not know if the original theory has this property or not, but we may introduce a new theory in which this property is now incorporated as an axiom. The new theory and the old may or may not be equivalent, in general we simply do not know, so this is a striking case of a neutral analogy between model and a theory being converted into a positive analogy. Alternatively where the model enriches a theory, some particular property may be made model-independent by fiat and fedback as an additional constraint on the old theory. Examples of both heuristic methods are given from the field of particle physics.

3. The testing of theories:

If a model is used to make empirical predictions a difficulty arises in attributing a falsification to the underlying theory of the special properties of the model, for example in the case of impoverishment whether the approximations made in arriving at the model are the source of the incorrect predictions. It is only if we have independent evidence for the truth of the theory that empirical tests of a model can be used to assess mathematical approximations. In practice physicists are well aware of this situation and in cases where approximations cannot be assessed directly the technique of 'modelling' a theory in two or more different ways, and comparing the results, is often employed as a method of assessing the accuracy of approximations. The Bayesian reasoning behind this approach is noted. The case of the 'floating' model is next discussed. Here the model is again disconnected from the theory by the so-called 'computation gap'. But also the predictions of the model do not match with observation. Nevertheless the model may be claimed to be of value. Professor Post's strictures on this situation are examined and it is concluded that floating models should only be employed where adjustment of the model to secure agreement with experiment can be made in an intuitively simple way. The adjusted model can then be thought to capture essential features of the underlying theory, insofar as this theory is itself believed to be true.

Finally it is argued that the employment of models in the sense of impoverished theories is essential to scientific method in general. It is moreover in this respect that the imaginative and intuitive element in theoretical physics is most clearly seen so far as normal science in the Kuhnian sense is concerned. Modelling is an Art and insofar as it is essential to Science the latter can itself be said to be more of an art than a science.

MODELS IN PHYSICS: ABSTRACT

The relevance of models in physics to philosophical problems lies solely in the light they throw on the problems of predictability and realism. Uses of the term 'model' in modern theoretical physics that have no bearing on these problems seem to result from the fact that physics has increasingly become an abstract game of symbols rather than a traditional type of empirical science.

The *problem of predictability* is this. Campbell argued, against such formalists as Duhem, that if theoretical systems are divorced from physical interpretation, except for a few peripheral translations into experimental laws, then their development would become scientifically arbitrary and subject to no constraints except those of pure mathematics and logic. Therefore, he argued, the theoretical axioms themselves must be interpretable into a model derived from some physical system *other than* the explanandum of the theory. He unfortunately used the term 'analogy' for this model, although 'analogy' is a relational term, and it was not clear what two relata the analogy was supposed to relate. My terminology of model_1, model_2 and positive, negative and neutral analogy was supposed to sort out this muddle. I argued further that Campbell was right to reject formalism as a philosophy of theoretical science for essentially the reasons he gave. I have since reinforced this argument along the lines of De Finetti's personalist probability theory, by suggesting that the relation of analogy is the fundamental inductive relation which grounds our instinctive confidence in all inductive predictions (see *The Structure of Scientific Inference*).

I argued against Campbell that for this notion of predictive confidence to work, some form of realism must be assumed. The *problem of realism* has since taken a new lease of life as a result of the debate about theorychange and meaning variance. The sequence of paradigms in their theoretical aspect may be equivalently described as a sequence of very general theoretical models or conceptual frameworks (*not* in Dr Redhead's sense of an "impoverishment of theory" - they are the *presuppositions* of theory languages). None of these theoretical models capture the "real", and in their historical sequence there is not even any discernible approximation to the ontology of the real (atoms and void give place to continuous fields of force, to discrete transitions between quantum states, to ... etc.). The question is, what is the relation of a given such conceptual scheme to physical reality? We have in any case to abandon naive billiard ball realism, what can be put in its place which will among other things be adequate for a theory of the predictive power of physics as explained above? Some realism of physically invariant mathematical structures seems indicated, provided physics still claims to be strongly predictive and not merely either instrumental or a non-empirical mathematical game. Modern physics is a crucial case for the interpretation of models, but its relation to the philosophical problems of model use remains to be worked out.

MARY HESSE

Models   Comments

1) Role of analogy in explaining how predictive success in new domains is possible.

If X resembles in property P and if Y has property Q

then it more than likely that X also has property Q. — Hesse. *Models, Analogies*

2) Role in explaining – explain the unfamiliar in terms of the familiar – Campbell etc.

3.) But I want to describe what scientists actually do. →

Models are inadequate theories about something we are interested in.

Analogue models are adequate theories about something we are *not* interested in.

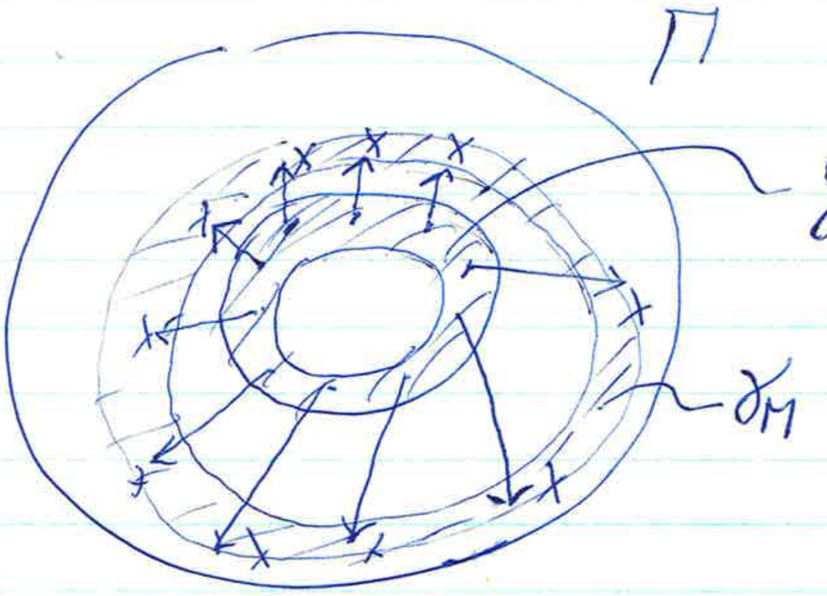
Weisskopf quote'

Models, approximations – General Treatment

$\Gamma = \{\psi(\xi)\}$ is space of all functions from $\{\xi\}$ into $\{\psi\}$ and all solutions of such functions.

γ = subset of members of Γ which are allowed by equations of motion of theory T

then in a model M or an approximation the individual points in γ are moved closer to points in Γ as illustrated



the end-points of all arrows (marked x) are elements of a new set γ_M which constitutes a model for T

Thus f is some function from set X or subset of X into set Y so we write $f: X \longrightarrow Y$
then $\Gamma = \{f\}$

Bayesian Analysis of Coincidence in Model Predictions

Let x = prior probability that any one model is adequate

Then assuming independence, probability that all m models are adequate is x^m and probability that all the models are inadequate is $(1-x)^m$

Now ~~coincidence~~ m-fold coincidence can only occur if all or none of the m models are adequate, so if ε is probability of coincidence if none of the models is adequate, Bayes's Theorem gives

$$P_m = \text{Prob}\left(\begin{matrix}\text{prediction}\\ \text{is true}\end{matrix}\,\middle|\,\begin{matrix}m\text{-fold}\\ \text{coincidence}\end{matrix}\right) = \frac{x^m}{x^m + \varepsilon(1-x)^m}$$

$$\varepsilon = \frac{10^n-1}{(10^n-1)^m} \simeq 10^{-n(m-1)}$$

where n is no of digits in prediction

(the -1 arises because we exclude the coincidence which is a true prediction)

now $P_1 = \dfrac{x}{x + 1-x} = x$ as we expect

and we are interested in an enhancement ratio

$$\Omega = \frac{P_m}{P_1} = \frac{1}{x + \frac{\varepsilon}{x^{m-1}}(1-x)^m}$$

$$\simeq \frac{1}{x + \varepsilon/x^{m-1}} \quad \text{if } x \text{ is small}$$

We now distinguish two cases (assuming $x \ll 1$)

(1) x^{m} ~~x^{m}~~ $\gg \varepsilon$ giving $R \simeq 1/x \gg 1$

so posterior probability P_m is very close to unity

(2) $x^{m-1} \ll \varepsilon$, giving $R \simeq \dfrac{x^{m-1}}{\varepsilon} \ll 1$

In the first case the coincidence has enhanced our expectation that the prediction is a true one, but in the second case the coincidence has actually increased our belief in the falsity of the prediction.

So the question of whether or not observing a coincidence in model predictions can be used to assess the accuracy of the prediction depends critically on our prior probability x.

To put in some figures with $n=3$, $m=3$,
~~$\varepsilon = 10^{-6}$~~ ~~and for $x = 10^{-1}$, $R = 9.99$~~
while ~~for $x = 10^{-4}$ $R = 0.01$~~
and ~~for $x = 10^{-3}$ $R = 1.00$~~

$\varepsilon = 10^{-6}$ and we have.

x	R	P_3
0.1	9.99	0.999
0.01	50.00	0.5
0.001	1.00	0.001
0.0001	0.01	0.000001

Our analysis shows that the method of coincidence in model predictions is not automatically a sound aid for assessing approximations, but

P.T.O.

depends on our having some reasonable
prior confidence in the adequacy of
the models.

Doyles Thm on Non-linear decomposition

1.) $\dot{x} = F_1(x) + F_2(x)$

Eg. 2 harmonic oscillators with freqs. ω_1, ω_2

$x = \sin(\sqrt{\omega_1^2+\omega_2^2})t \neq \sin\omega_1 t + \sin\omega_2 t$

2.) damping, oscillators as constituents in motion of damped harmonic oscillator

3.) Fourier analysis — ~~linear~~ math. significance only.

4.) Vector decomposition: not specially confined to linear theories.

But in Q.M. superposition is preserved in reality.

Bee
7.14

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Now m-fold coincidences can only occur if all or none of the m models are adequate, so if ε is probability of coincidence if none of the models is adequate, Bayes's theorem gives

$$P_m = \text{Prob}\left(\begin{matrix}\text{Model}\\ \text{prediction is}\\ \text{accurate}\end{matrix} \middle/ \begin{matrix}m\text{-fold}\\ \text{Coincidence}\end{matrix} \right) = \frac{x^m}{x^m + \varepsilon(1-x)^m}$$

$$\varepsilon = \frac{10^n - 1}{(10^n-1)^m} \simeq 10^{-n(m-1)}$$

where n is number of digits in prediction

(The minus one arises because we exclude the coincidence which agrees with the prediction of the theory T)

Now $P_1 = \dfrac{x}{x+1-x} = x$ as we expect

and we are interested in an enhancement ratio

$$R = \frac{P_m}{P_1} = \frac{1}{x + \frac{\varepsilon}{x^{m-1}} \cdot (1-x)^m}$$

$$\simeq \frac{1}{x + \frac{\varepsilon}{x^{m-1}}} \quad \text{if } x \text{ is small}$$

We now distinguish two cases (assuming $x \ll 1$)

(1) $x^m \gg \varepsilon$, giving $r \simeq 1/x \gg 1$
so posterior probability P_m is very close to unity.

(2) $x^{m-1} \ll \varepsilon$, giving $r \simeq \frac{x^{m-1}}{\varepsilon} \ll 1$

In the first case the coincidence has enhanced our expectation that the model prediction is adequate, but in the second case the coincidence has actually increased our belief in the *inadequacy* of the prediction.
Our analysis shows that the method of coincidence in model predictions is not *automatically* a sound one for assessing approximations, but depends on our having some reasonable prior confidence in the adequacy of the models.

Interpretation in QM

1.) F.D. Statistics

H. 295 We cannot distinguish particles → non-individuality

But symmetry postulates are tested ones cf Messiah, Greenberg around 1963.

All Q's commute with all P's, so Q are symmetric functions of the labels.

Hence ~~$(P\psi, Q P\psi)$~~ $\langle P\psi | Q | P\psi \rangle$
$= \langle \psi | Q | \psi \rangle$.

$P^2 = 1 \quad P = P^{-1} \quad , P\psi = \lambda\psi$

$P = P^{*} \quad P^2\psi = \lambda P\psi = \lambda^2 \psi = \psi \quad \lambda^2 = 1 \quad \lambda = \pm 1$

But is $P\psi = \pm\psi$ if true for one dimensional symmetric, only possible reps of symmetric group. Introduce I. rep. as empirical condition then we stay always in same irr. rep. → cf. G = Gos —

2.) Hole Theory of Positrons

H. 295. negative energy states refute QM — loss of 1 particle theory.
D's hole theory is a many body theory i.e. hole theory has an existential commitment — there are ∞ many particles in certain states — same situation as in theory of hydrogen atom. Even in most detailed theory we want in order to apply the theory add the existential hypothesis that objects or systems exist to which the theory is applicable.

→ componentes

constantes fondos

~~$P_1 \dots P_n = Q_1 \dots Q_n \; (x_1 \dots x_n)$~~

$P_1 \dots P_n = x_1 \dots x_n \; (Q_1 \dots Q_2)$

$X = X \; (X)$

Selore $X = a_1 Q_1 + a_2 Q_2 + \dots$

o $X = X \; (F)$